Current Waveforms for Lightning Simulation

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***Abstract*—From a survey of the literature on lightning characteristics, we present and discuss our recommendations for the median (50%) and severe (1%) values of the salient parameters of lightning current in both positive and negative cloud-to-ground discharges. We present general expressions, suitable for numerical simulation of lightning effects, for lightning current versus time, current derivative versus time, second current derivative versus time, charge transfer versus time, and action integral (specific energy) versus time. We give sets of constants for these expressions such that the resultant waveforms for positive and negative flashes and for their component strokes and continuing current exhibit approximately the median and severe lightning current parameters recommended, and otherwise resemble the measured waveforms found in the literature.**

***Index Terms*—Atmospheric electricity, lightning, lightning current, lightning parameters, lightning simulation.**

I. INTRODUCTION

DIRECT effects of lightning current injection (e.g., direct-strike overvoltages on overhead power lines and damage to metallic surfaces) and induced effects from nearby lightning (e.g., induced voltages and currents in the internal electronics of lightning-struck aircraft, on electronics and explosive materials contained inside various other imperfectly shielded enclosures, and on power and communication lines) are often evaluated by numerical simulation involving assumed lightning current waveforms [1]–[16]. The current waveforms used in these simulations are sometimes simple double-exponential functions, expressed by the difference of two exponentials that decay differently with time [1], but more commonly in the recent literature are single or multiple Heidler functions [17]. The current waveforms typically used approximate a severe lightning current, whose characteristics are specified in standards such as [18]–[21].

We have surveyed the lightning literature to determine the median (50%) and severe (1%) values of the salient characteristics of lightning currents at ground for full flashes and for their first and subsequent component strokes and continuing current, considering flashes transferring either negative or positive charge between cloud and ground. We present and discuss our view of these median and severe values and then give expressions suitable for numerical simulation of lightning current versus time, first and second current derivatives versus time, charge transfer versus time, and action integral (specific energy) versus time that 1) exhibit the median and severe values recommended and 2) resemble the measured waveforms found in the literature. We suggest that the expressions presented could profitably be used in the future numerical simulations of the effects of lightning currents on objects and systems.

De Conti and Visacro [22] have published a study closely related to that presented here. They employ Heidler functions to model the median rise-to-peak characteristics of first and subsequent negative return-stroke currents observed on towers on Mount San Salvatore, Switzerland, and at Morro do Cachimbo, Brazil. We extend that study and similar studies (e.g. [2] and [23]–[25]) of negative return-stroke current waveform characteristics to include continuing current and full-flash charge transfer and action integral (specific energy) for both positive and negative flashes, for both median and severe cases.

II. RECOMMENDED MEDIAN AND SEVERE LIGHTNING PARAMETERS

The probability distribution functions of some lightning parameters have been shown from measured data to be approximately log-normal [26]. For an assumed log-normal distribution, knowledge of the median (50%) and severe (1%) values is sufficient to define the entire distribution. Six important lightning parameters that have been demonstrated to follow the log-normal distribution to a reasonable degree of approximation are the negative first and subsequent return-stroke peak currents [27], the charge transfer to 1 ms for negative first and subsequent return strokes [27], positive first return-stroke peak current [27], and the time interval between negative strokes [28]. Cianos and Pierce [29], in a table reproduced in [26], list 10 lightning parameters that they suggest can be described satisfactorily by a log-normal distribution: flash duration, interstroke interval, return-stroke peak current, flash charge transfer, time to return-stroke current peak, rate of rise of return-stroke current, time to return-stroke current half value, duration of continuing current, continuing current amplitude, and continuing current charge. Nevertheless, some of these parameters are only crudely approximated by the log-normal distribution, and those are certainly not described satisfactorily enough by that distribution to allow adequate prediction of extreme values.

Table I contains our recommendations for median and severe parameters characteristic of cloud-to-ground lightning from our review of the literature. Negative first and subsequent strokes, positive first strokes, negative and positive continuing currents, and negative and positive flashes are treated separately. All

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TABLE I
MEASURED LIGHTNING CURRENT CHARACTERISTICS RECOMMENDED BY THE PRESENT STUDY AND MODELED VALUES

	Measured Values		Modeled Values	
	50%	1%	50%	1%
RETURN STROKE PARAMETERS				
NEGATIVE FIRST STROKES				
(a) Peak current (kA)	30	150	32	160
(b) Time from zero to current peak (μs)	5	30	6	6
(c) Maximum rate of current rise (kA/μs)	100	400	100	500
(d) Time to decay from peak to half-peak value (μs)	70-80	300	75	80
(e) Charge Transfer (C)	5	40	5.5	27
POSITIVE FIRST STROKES				
(a) Peak current (kA)	35	500	35	350
(b) Time to current peak (μs)	10-20	150	11	11
(c) Maximum rate of current rise (kA/μs)	100	400	100	500
(d) Time to decay to half-peak value (μs)	†	†	30	40
NEGATIVE SUBSEQUENT STROKES				
(a) Peak current (kA)	10-15	50	11	56
(b) Time to current peak (10-90 percent) (μs)	0.3-0.6	9	0.6	0.6
(c) Maximum rate of current rise (kA/μs)	100	400	97	490
(d) 10 to 90 percent rate of current rise (kA/μs)	30-50	150	77	77
(e) Time to decay from peak to half-peak value (μs)	30-40	250	35	35
NEGATIVE CONTINUING CURRENT LONGER THAN 40 ms				
(a) Amplitude (A)	100-200	1000	200	1000
(b) Duration (ms)	100-200	600	200	200
(c) Charge transfer (C)	10-20	200	14	69
POSITIVE CONTINUING CURRENT				
(a) Amplitude (kA)	1-5	10-30	4	30
(b) Duration (ms)	85	1000	85	85
(c) Charge transfer (C)	80	700	120	890
NEGATIVE FLASH PARAMETERS				
(a) Number of strokes	3-5	25	3	15
(b) Interstroke interval (ms)	60	600	*	*
(c) Duration (ms)	200	1000	*	*
(d) Charge transfer (C)	20	200	25	280
(e) Action integral (A^2·s)	8x10^4	3x10^6	9x10^4	5x10^6
POSITIVE FLASH PARAMETERS				
(a) Number of strokes	1	3††	1	1
(b) Duration (ms)	85	1000	85	85
(c) Charge transfer (C)	80	700	120	910
(d) Action integral (A^2·s)	7x10^5	6x10^7	3x10^5	2x10^7

† See discussion in Section II (E).

†† Generally separate channels to ground, so for direct current injection there is effectively only one stroke.

* Not modeled.

parameters listed in Table I represent lightning between the cloud and ground as observed near the ground, where the return-stroke currents exhibit their highest peak values. The characteristics of intracloud lightning, intercloud lightning, and cloud-to-ground lightning far above the ground are much less well studied than cloud-to-ground flashes near the ground, but are generally thought to be less severe.

There is no consensus in the literature as to what statistical value constitutes a severe case. Many investigators have adopted the 5% values of [27]. In this paper, 1% was chosen by the sponsor of this study. Perhaps, "extreme" is a better description of the 1% value than "severe." The presentation of the second derivative of the return-stroke waveforms given in this paper is seldom found in the literature, but was required by the sponsor for its specific lightning current simulation.

We comment in the following on the choice of the parameters listed in Table I.

A. Return-Stroke Peak Current

The peak current data in Table I for positive first strokes (rarely are there positive subsequent strokes—see positive flash parameters in Table I) and for first and subsequent negative strokes are taken from [27] and their referenced previous work. The median (50%) values are relatively well established, and the 1% values are chosen from fitting log-normal distributions to the measured data, although some experimental data near the 1% values of the data-fitting curve are available.

B. Maximum Rate of Return-Stroke Current Rise and Other Rise-to-Peak Characteristics

In tower measurements, such as those presented in [27], this parameter is likely underestimated because of measurement system limitations and the potential influence of the strike object. Schoene *et al.* [30] have shown that the strike object can affect rise-time parameters and that the highest rate of rise is for a relatively small, well-grounded object. The value of 100 kA/μs, adapted as the 50% maximum rate of rise for both positive strokes and for negative first and subsequent strokes, has been measured on well-grounded strike objects for negative strokes in triggered lightning, those strokes being similar, if not identical, to subsequent strokes in natural negative lightning [30]–[32].

The inference that the same 50% maximum rate of rise of current characterizes negative and positive first strokes, as is measured for negative subsequent strokes, follows from the observation that the maximum rate of change of the remote electric field for the three types of return strokes over salt water (a relatively good conductor) is essentially the same [33]–[35]. The 1% maximum rate of rise of 400 kA/μs listed in Table I is near the largest value measured for a triggered-lightning return stroke, 411 kA/μs [36], and the largest value measured for lightning interaction with an aircraft in flight, 380 kA/μs [37].

Return-stroke current rise-time characteristics such as time to peak and 10 to 90% rise-time are determined from measured triggered-lightning current waveforms and tower current waveforms (primarily [27]), with comparison of the measured current characteristics to electric field and electric field derivative measurements for lightning over salt water being used to infer current characteristics not adequately measured directly [33], [34], [38]. Additionally, the full-width at half-maximum of the initial half cycle of the current derivative is a measure of the current rise time; and the full-width at half-maximum of the second derivative of current is a measure of the duration of the high current derivative, an important parameter in cases of inductive-circuit flashover [38].

C. Flash Charge Transfer

The charge transfer values in Table I are taken primarily from the experimental data of [27] and log-normal distribution fits to those data. For a positive flash, 700 C is inferred from the log-normal distribution fit as the 1% value, whereas the largest measured value in [27] is 400 C at the 4% level. There have been a number of direct tower measurements of both positive and negative charge transfer between 300 and 1000 C for lightning in Japanese winter storms, with one positive charge transfer reported to exceed 3000 C [39], [40]. These charge transfers may not be from cloud-to-ground flashes containing return strokes and initiated in the cloud charge, but may rather be from upward-initiated lightning. Nevertheless, extreme current waveform statistics should include those salient characteristics of the rarer upward-initiated lightning. Positive charge transfers inferred from remote magnetic fields in [41] and [42] are up to roughly 2000 to 3000 C. The International Standard IEC 62305-1,3:2006 [18], [19] lists 300 C as a “severe” charge transfer for all types of flashes.

D. Flash Action Integral

The values for action integral in Table I are taken from the data of [27], references to their previous work given in that paper, and log-normal distribution extrapolations of those measurements. It is often difficult to decide when a return-stroke current ends and a continuing current begins, particularly for positive flashes, which almost always exhibit large, long-duration slowly-varying currents following an initial current peak. If such long-duration currents are attributed to continuing current, then it is the continuing current that makes the major contribution to the flash action integral value (and to the charge transferred). It follows that the “time to decay to half-peak value” is not well defined for positive first strokes and hence not specified in Table I, and the peak current assigned to the positive continuing current is also to some extent arbitrary (see subsection IV-A in the following). The International Standard IEC 62305-1,3:2006 [18], [19] gives 10^7 $\mathrm{A}^2{\cdot}\mathrm{s}$ for a “severe” first-stroke action integral, whereas we give 6×10^7 $\mathrm{A}^2{\cdot}\mathrm{s}$ in Table I for the 1% value for a positive flash, consistent with the data of [27].

E. Continuing Current, Negative and Positive

Duration data for negative continuing current longer than 4 ms taken from the high-speed video measurements of [43] indicate that 15 ms is at the 50% level and 550 ms is at the 1% level. Kitagawa *et al.* [44] report that nearly half of about 200 negative ground flashes they studied exhibited a continuing current interval exceeding 40 ms and one quarter of all interstroke intervals contained such currents. Kitagawa *et al.* [44] term continuing currents exceeding 40 ms as “long continuing current.” In Table I, we present values only for long continuing currents. Duration data from electric field records and video observations reported in [45] indicate that the median negative long continuing current duration is near 200 ms. Their median duration for four positive long continuing currents is near 150 ms and their maximum duration is near 200 ms. Berger *et al.* [27] give 85 ms for the median duration of a positive flash and 500 ms for the 5% value, with both durations being predominantly the positive continuing current duration. Campos *et al.* [43], [46] subdivide the waveshapes of negative and positive continuing current into six waveshape types; 24% of both negative and positive continuing current exhibit type-1 behavior, “a more or less exponential decay” (originally identified for negative triggered-lightning continuing currents in [32]), the waveshape we will adopt for the modeling of continuing current of either polarity. The median and severe magnitudes of positive continuing current are not well studied. Some evidence for the values given in Table I is discussed in [47].

F. Other Parameters

The best overall discussion of the parameters not discussed in this section, for which there would not be much argument and which are not particularly critical to induced or direct lightning effects, is found in [47].

III. TIME-DOMAIN WAVEFORMS FOR FIRST STROKES, SUBSEQUENT STROKES, AND CONTINUING CURRENT

General time-domain expressions with unspecified constants are presented in the following for various current-related parameters, an approach suggested by De Conti and Visacro [22], following [17]. Constants are then chosen for these general expressions by trial and error (and are, in general, different from those found in previous publications) so that the waveforms approximate the various 50% parameters listed in Table I. The resultant waveforms can be considered typical. The current-related waveforms can be altered by changing the constants found in the functional expressions. Following the discussion of typical waveforms, we suggest constants to replicate severe waveforms.

General functional expressions for the return stroke and continuing current waveforms are given in the following, constructed of multiple Heidler functions [17]. There are four constants (I_{0k}, n_k, τ_{1k}, and τ_{2k}) for each term in the summations. The constant I_{0k} controls the amplitude, n_k controls the initial waveform steepness, τ_{1k} is the front-time constant, τ_{2k} is the decay-time constant, and η_k is termed the amplitude correction factor.

A. Current

$$i(t) = \sum_{k=1}^{m} \frac{I_{0k}}{\eta_k} e^{-t/\tau_{2k}} \frac{(t/\tau_{1k})^{n_k}}{1+(t/\tau_{1k})^{n_k}} \tag{1}$$

with

$$\eta_k = e^{-\tau_{1k}/\tau_{2k}(n_k(\tau_{2k}/\tau_{1k}))^{1/n_k}}.$$

B. Current Derivative

$$\begin{aligned} i'(t) = \sum_{k=1}^{m} - &\frac{e^{-t/\tau_{2k}}(t/\tau_{1k})^{n_k} I_{0k}}{t[(t/\tau_{1k})^{n_k}+1]^2 \eta_k \tau_{2k}} \\ &\times \left[t - n_k \tau_{2k} + t \left(\frac{t}{\tau_{1k}} \right)^{n_k} \right]. \end{aligned} \tag{2}$$

C. Current Second Derivative

$$\begin{aligned} i''(t) = \sum_{k=1}^{m} - &\left[\frac{e^{-t/\tau_{2k}}(t/\tau_{1k})^{n_k} I_{0k}}{t^2[(t/\tau_{1k})^{n_k}+1]^3 \eta_k (\tau_{2k})^2} \right] \\ &\times \left[n_k(\tau_{2k})^2 - 2t^2 \left(\frac{t}{\tau_{1k}} \right)^{n_k} - t^2 \right. \\ &- n_k^2(\tau_{2k})^2 - t^2 \left(\frac{t}{\tau_{1k}} \right)^{2n_k} \\ &+ 2tn_k\tau_{2k} + \left(\frac{t}{\tau_{1k}} \right)^{n_k} n_k(\tau_{2k})^2 \\ &+ \left(\frac{t}{\tau_{1k}} \right)^{n_k} n_k^2(\tau_{2k})^2 \\ &\left. +2t \left(\frac{t}{\tau_{1k}} \right)^{n_k} n_k\tau_{2k} \right]. \end{aligned} \tag{3}$$

D. Charge Transferred

$$\int i(t)dt = \int \sum_{k=1}^{m} \frac{I_{0k}}{\eta_k} e^{-t/\tau_{2k}} \frac{(t/\tau_{1k})^{n_k}}{1+(t/\tau_{1k})^{n_k}} dt. \tag{4}$$

TABLE II
CALCULATED VALUES OF HEIDLER FUNCTION PARAMETERS FOR A MEDIAN NEGATIVE SUBSEQUENT-STROKE CURRENT

k	I_{0k} (kA)	n_k	τ_{1k} (μs)	τ_{2k} (μs)
1	7.5	55	1.1	15
2	5	2	1.2	500

TABLE III
CALCULATED VALUES OF HEIDLER FUNCTION PARAMETERS FOR A MEDIAN NEGATIVE FIRST-STROKE CURRENT

k	I_{0k} (kA)	n_k	τ_{1k} (μs)	τ_{2k} (μs)
1	3	2	4	20
2	3	3	4	20
3	3	9	4	20
4	3	11	4	20
5	20	85	4.5	23
6	15	2	20	240

TABLE IV
CALCULATED VALUES OF HEIDLER FUNCTION PARAMETERS FOR A MEDIAN NEGATIVE CONTINUING CURRENT

I_{0k} (A)	n_k	τ_{1k} (μs)	τ_{2k} (μs)
200	9	4	70

TABLE V
CALCULATED VALUES OF HEIDLER FUNCTION PARAMETERS FOR A MEDIAN POSITIVE FIRST-STROKE CURRENT

k	I_{0k} (kA)	n_k	τ_{1k} (μs)	τ_{2k} (μs)
1	20	116	6	50
2	5	3	9	20
3	6	2.5	8	30
4	7	4	6	30

TABLE VI
CALCULATED VALUES OF HEIDLER FUNCTION PARAMETERS FOR A MEDIAN POSITIVE CONTINUING CURRENT

I_{0k} (kA)	n_k	τ_{1k} (μs)	τ_{2k} (μs)
4	9	3	30

E. Action Integral

$$\int i^2(t)dt = \int \left[\sum_{k=1}^{m} \frac{I_{0k}}{\eta_k} e^{-t/\tau_{2k}} \frac{(t/\tau_{1k})^{n_k}}{1+(t/\tau_{1k})^{n_k}} \right]^2 dt. \tag{5}$$

Equations (4) and (5) contain integrals, whose limits are the times at which current starts and stops flowing.

IV. MODELED MEDIAN AND SEVERE LIGHTNING FLASHES

Modeled flash component parameters obtained from all plotted waveforms are included in the two rightmost columns in Table I. These values are determined by examining the waveforms produced with input parameters given in Tables II–VI. All calculated charge transfer and action integral values from the models are approximate in that the analytical waveform is either terminated at 1 ms (return stroke) or at 7% of its peak value after the peak (continuing current) in the modeling.

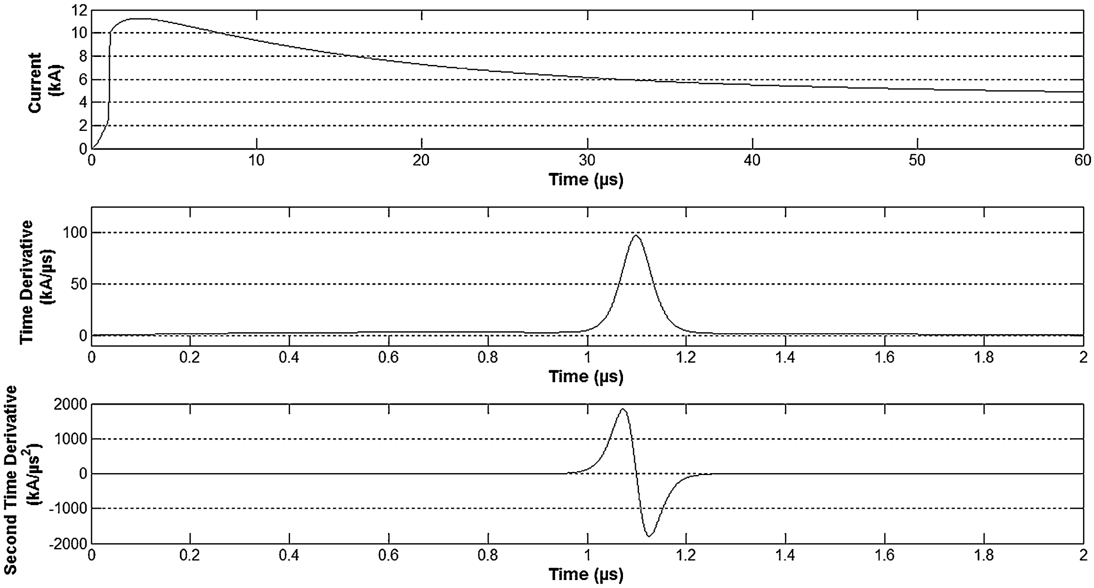

Fig. 1. Modeled current waveform for a median negative subsequent stroke: (a) current versus time; (b) first time derivative of current; (c) second time derivative of current. Note that the waveform in (a) is shown on a 60-μs time scale, while those in (b) and (c) are on a 2-μs time scale.

A. Median (50%) Negative Flashes

We first examine the negative subsequent stroke because it is the simplest to represent. To synthesize accurately a typical negative subsequent-stroke waveform, only two Heidler functions are necessary, i.e., $m = 2$ in the previous expressions. The values for the four adjustable parameters are given in Table II, with plots of the current, the first derivative of the current, and the second derivative being shown in Fig. 1. For the negative subsequent-stroke constants given in Table II, the negative subsequent-stroke charge transfer is 2.4 C and the action integral is 8.5×10^3 A^2·s.

To represent a negative first-stroke waveform with a single initial peak, we sum six Heidler functions ($m = 6$). The parameters used for each Heidler function are given in Table III. De Conti and Visacro [22] have shown that a double-peaked first-stroke current matching observations at Mount San Salvatore and Morro do Cochimbo can be obtained by adding a seventh Heidler function to the six of Table III, but this addition does not significantly change the median parameters. On the other hand, Silveira *et al.* [2] argued that the double-peaked representation leads to a 25% increase in the calculated power line overvoltages due to direct strikes and a similar decrease for induced voltages. Nevertheless, we sum six Heidler functions to preserve the common single-peaked waveform for simulation and analysis, while reproducing the essential complexities of a first-stroke waveform front, as described in the next paragraph. Fig. 2 shows the single-peaked current, the first derivative of the current, and the second derivative of the current.

The negative first-stroke current rise time to peak consists of a concave "slow front" of some microseconds followed by a "fast transition" of tenths of a microsecond to the peak value during which the maximum current derivative occurs, as observed on towers in Switzerland [27], Brazil [2], [22], and elsewhere, and as inferred from electric field measurements (e.g., see [48]). For the first-stroke constants given, the negative first-stroke charge transfer is 5.5 C and the action integral is 7.1×10^4 A^2·s.

Willett and Krider [38] report mean full-width half-maximum values for *dE*/*dt* measurements of first and subsequent strokes to be 79 and 72 ns, respectively. These values are to be compared with the values from Figs. 1(b) and 2(b), 190 ns and 72 ns, respectively, from our modeled currents.

One Heidler function is used to reproduce the waveform (see Fig. 3) for the negative continuing current. The values for the parameters are summarized in Table IV. The continuing current charge transfer is 14 C and action integral is 1.5×10^3 A^2·s.

Since the actual continuing current merges smoothly with the tail of the return-stroke current, the continuing current rise-time characteristics given in Table IV and Fig. 3 are nonphysical and arbitrary. One should envision the return-stroke current tail decreasing in such a manner that the addition of the continuing current waveform of Fig. 3 results in a relatively smooth overall current waveform at the start of the continuing current.

For a median (50%) negative flash with parameters described in Table I, we assume that the flash is composed of a median first stroke, two median subsequent strokes (although Table I specifies 3–5), and a median continuing current. As noted earlier, we assume that all stroke currents end at 1 ms and the continuing current ends at 7% of its peak value. The calculated total charge transfer is 25 C and the total action integral is 9×10^4 A^2·s, to be compared to the measured values of 20 C and 8×10^4 A^2·s.

B. Severe (1%) Negative Flashes

Functional expressions have been given previously for currents in negative first and subsequent strokes and continuing current, and constants have been chosen to reproduce the median parameters found in Table I. To simulate a reasonable approximation to a negative severe flash, we assume that the flash is composed of a severe first stroke, 15 severe subsequent strokes

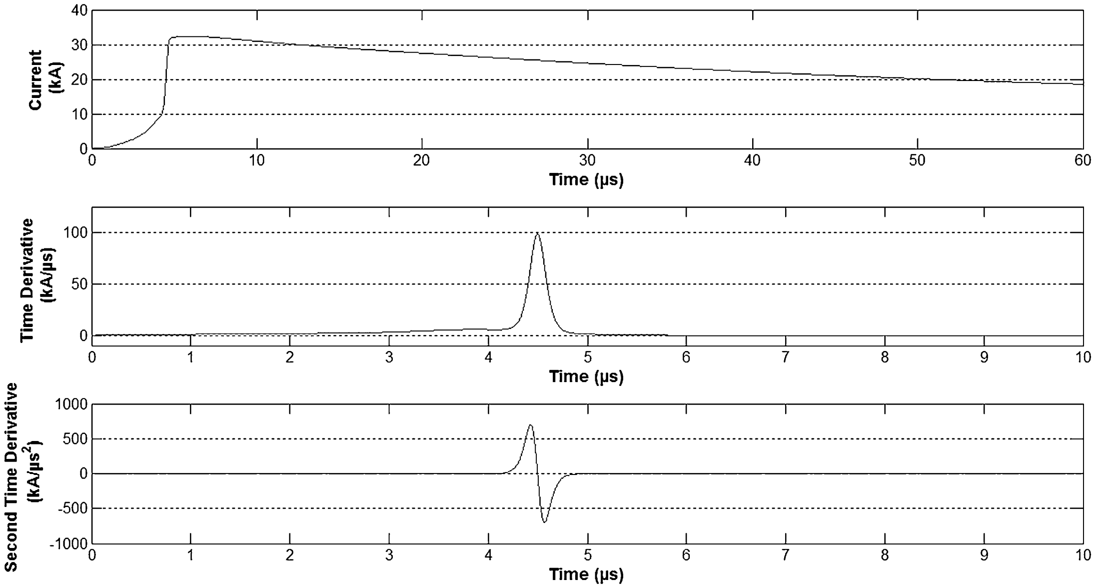

Fig. 2. Modeled current waveform for a median negative first stroke: (a) current versus time; (b) first time derivative of current; (c) second time derivative of current. Note that the waveform in (a) is shown on a 60-μs time scale, while those in (b) and (c) are on a 10-μs time scale.

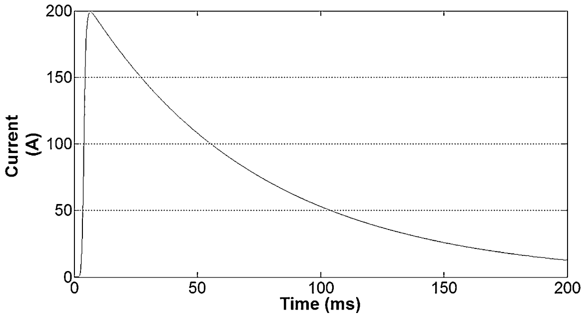

Fig. 3. Modeled current waveform for a median negative continuing current. See comments in subsection IV-A.

(although Table I specifies 24), and a continuing current with median duration and severe amplitude.

To accomplish this, we multiply the first and subsequent return-stroke median amplitude factors by five to ensure a peak current approximately five times the median (50%) negative flash peak current, and multiply the continuing current amplitude factor by five to obtain a severe peak continuing current of 1 kA. The I_{0k} values for the severe first and subsequent return strokes, as well as for the severe continuing current are obtained by multiplying the I_{0k} values in Tables II–IV by the multipliers indicated previously (five), with all other constants in the Tables kept at the median values. We calculate a total flash charge transfer of 280 C and an action integral of 5×10^6 $A^2 \cdot s$ from our models, to be compared to measured values of 200 C and 3×10^6 $A^2 \cdot s$ given in Table I.

Alternatively, we could use two severe subsequent strokes with a severe continuing current duration and amplitude, along with a τ_{2k} value of 220 ms. In this case, the charge transfer would be 260 C and the action integral 2.4×10^6 $A^2 \cdot s$.

Clearly, we can obtain the severe values in Table I for negative flash charge transfer and action integral in more than one way (by varying the continuing current duration or the number of subsequent strokes).

For the severe negative first and subsequent strokes outlined here, the maximum rate of current rise is 500 kA/μs, comparable to the 400 kA/μs found in Table I.

C. Median (50%) Positive Flashes

To represent a positive first-stroke waveform, we sum four Heidler functions ($m = 4$). The parameters used for each Heidler function are given in Table V. Fig. 4 shows the current, first derivative of the current, and the second derivative of the current. The positive first-stroke charge transfer is 1.7 C and the action integral is 3.3×10^4 $A^2 \cdot s$. Recall that these values are to be added to the corresponding values for continuing currents in order to obtain realistic overall waveforms.

One Heidler function is used to produce the waveform (see Fig. 5) for the positive continuing current. The values for the parameters are summarized in Table VI. We take the peak current to be 4 kA in magnitude. Because so little is known about the positive flash continuing current, a 4-kA peak compared with a 35-kA stroke peak current does not appear unreasonable. The charge transfer is 120 C and the action integral is 3×10^5 $A^2 \cdot s$, to be compared to the measured values of 80 C and 7×10^5 $A^2 \cdot s$ in Table I. As with the negative continuing current, the rise time of the continuing current is nonphysical and arbitrary (see previous discussion of negative continuing current).

D. Severe (1%) Positive Flashes

To simulate the peak currents of a positive flash having parameters near their 1% values, we multiply the first-stroke amplitude factors (I_{0k}) by 10 to ensure a peak current approximately 10 times the median (50%) positive flash peak current, and multiply the continuing current amplitude factor by 7.5 to obtain a severe peak continuing current of 30 kA. The I_{0k} values for the severe

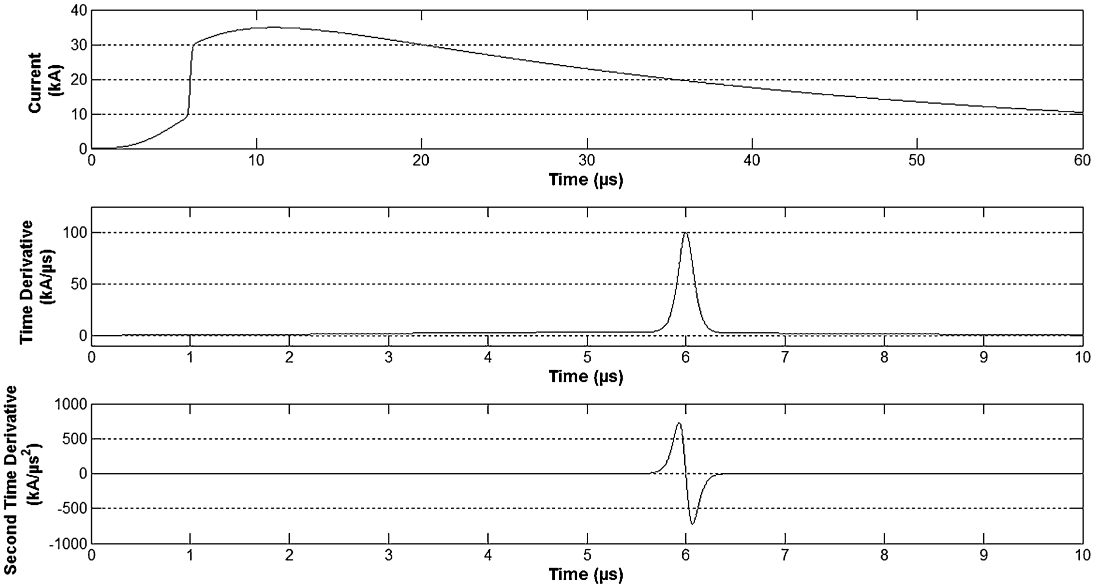

Fig. 4. Modeled current waveform for a median positive first stroke: (a) current versus time; (b) first time derivative of current; (c) second time derivative of current. Note that the waveform in (a) is shown on a 60-μs time scale, while those in (b) and (c) are on a 10-μs time scale.

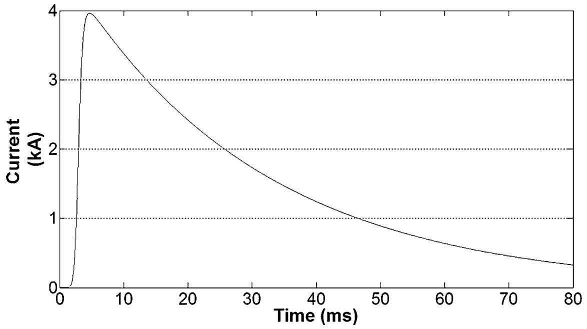

Fig. 5. Modeled current waveform for a median positive continuing current.

first-return strokes as well as for the severe continuing current are obtained by multiplying the I_{0k} values in Tables V and VI by the multipliers indicated previously (10 and 7.5, respectively), with all other constants in the tables kept at the median values. Using a continuing current waveform with 30-kA peak current amplitude and 85-ms duration, we obtain a total flash action integral of 2×10^7 A^2·s, which is comparable to the value of 6×10^7 A^2·s given in Table I. The charge transfer is near 910 C, while 700 C is given in Table I. A severe charge transfer near 1000 C is not unreasonable since charge transfers of this magnitude (and greater) have been measured in winter storms in Japan [39], [40] and inferred in [42]. Additionally, using a continuing current waveform with peak amplitude of 30 kA appears reasonable if the positive stroke peak amplitude is near 350 kA. The calculated maximum rate of current rise for a positive severe flash with these parameters is almost 1000 kA/μs, while it is 400 kA/μs in Table I.

Alternatively, to obtain a different but reasonable severe positive flash action integral and charge transfer, one could use a continuing current waveform with 10-kA peak amplitude and 1000-ms duration (a τ_{2k} value of 350 ms), along with the severe return stroke described previously. The total action integral would be 2.1×10^7 A^2·s and the total charge transfer would be 3300 C, both larger than the experimental values in Table I, but not unreasonable.

V. CONCLUDING REMARKS

We have extended previous studies on lightning current waveform simulation by including flash charge transfer and total action integral values for positive and negative median and severe flashes. Calculated median (50%) and severe (1%) constants (see Tables II–VI) for all flashes were chosen in order to approximate these two important parameters, as well as other lightning characteristics, so as to reproduce the measured values found in the literature and summarized in Table I. While it is possible to simulate either measured flash charge or measured action integral exactly, the other parameter will then not necessarily be well simulated; therefore, compromises are made to simulate both to reasonable approximation.

In order to approximate some of the severe (1%) calculated values for flashes, we arbitrarily chose specific parameters, with the result that a severe flash does not include all severe component values. If each severe component value were to occur for a given flash, then the flash likely would become more extreme than the severe (1%) case. For example, to simulate a severe negative flash, we did not use the 24 subsequent strokes, as indicated for a severe flash in Table I, but instead chose 15 strokes. Having 24 subsequent strokes is very rare, and having each with approximately 50-kA peak current does not often (or ever) happen. The number of strokes chosen will change the charge transfer and action integral linearly.

For both severe positive and severe negative flash continuing currents, our simulations used the median value for duration, but severe values for peak currents. We decided that it would be too extreme to have continuing currents with severe peak currents as well as severe durations, and chose to have the severe peak current, while maintaining the median duration. Clearly, there are other possibilities that give similar action integrals and charge transfers.

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REFERENCES

[1] J. Chen, B. Zhou, F. Zhao, and S. Qiu, "Finite-difference time-domain analysis of the electromagnetic environment in a reinforced concrete structure when struck by lightning," *IEEE Trans. Electromagn. Compat.*, vol. 52, no. 4, pp. 914–920, Nov. 2010 (doi:10.1109TEMC.2010.2043437).

[2] F. H. Silveira, A. De Conti, and S. Visacro, "Lightning overvoltage due to first strokes considering a realistic current representation," *IEEE Trans. Electromagn. Compat.*, vol. 52, no. 4, pp. 929–935, Nov. 2010 (doi:10.1109/TEMC2010.2044042).

[3] M. P. Perkins, M. M. Ong, E. W. Crull, and C. G. Brown, Jr., "Modeling techniques used to analyze safety of payloads for generic missile type weapons systems during an indirect lightning strike," in *Proc. 31st Annual Antenna Meas. Tech. Assoc. Symp.*, Salt Lake City, UT, November 1–November 6, 2009.

[4] M. Aprà, M. D'Armore, K. Gigliotti, M. S. Sarto, and V. Volpi, "Lightning indirect effects certification of a transport aircraft by numerical simulation," *IEEE Trans. Electromag. Compat.*, vol. 50, no. 3, pp. 513–523, Aug. 2008 (doi:10.1109/TEMC.2008.927738).

[5] M. Aprà, M. D'Amore, M. S. Sarto, and V. Volpi, "Prediction of indirect lightning effects on a metallic-composite aircraft," , presented at the 39th AIAA Sci. Meet. Exhib., Reno, NV, Jan. 8–11, 2000, Paper AIAA 2001-0548.

[6] M. Aprà, M. D'Amore, M. Feliziani, F. Maradei, M. S. Sarto, A. Scarlatti, and V. Volpi, "Lightning stroke to a metallic-composite aircraft: Certification feasibility by simulation—Part II: Prediction of the induced electromagnetic field," in *Proc. 4th Int. Symp. Electromagn. Compat.*, vol. 2, Brugge, Belguim, pp. 179–184.

[7] I. A. Metwally and F. H. Heidler, "Reduction of lightning-induced magnetic fields and voltages inside struck double-layer grid-like shields," *IEEE Trans. Electromagn. Compat.*, vol. 50, no. 5, pp. 905–912, Nov. 2008.

[8] I. A. Metwally and F. H. Heidler, "Computation of transient overvoltages in low-voltage installations during direct strikes to different lightning protection systems," *IEEE Trans. Electromagn. Compat.*, vol. 49, no. 3, pp. 602–613, Aug. 2007.

[9] I. A. Metwally, F. H. Heidler, and W. J. Zischank, "Magnetic fields and loop voltages inside reduced-and full-scale structures produced by direct lightning strikes," *IEEE Trans. Electromagn. Compat.*, vol. 48, no. 2, pp. 414–426, May 2006.

[10] I. A. Metwally, W. J. Zischank, and F. H. Heidler, "Measurement of magnetic fields inside single- and double-layer reinforced concrete buildings during simulated lightning currents," *IEEE Trans. Electromagn. Compat.*, vol. 46, no. 2, pp. 208–221, May 2004.

[11] Y. Baba and V. A. Rakov, "Voltages induced on an overhead wire by lightning strikes to a nearby tall grounded object," *IEEE Trans. Electromagn. Compat.*, vol. 48, no. 1, pp. 212–224, Feb. 2006 (doi: 10.1109/TEMC.2006.870807T).

[12] J. Clancy, C. G. Brown, M. M. Ong, and G. A. Clark, "Lightning protection certification for high explosives facilities at Lawrence Livermore National Laboratory,", presented at the IEEE Antennas Propag. Soc. Conf., Albuquerque, NM, Jun. 19–23, 2006.

[13] A. Kern, F. Heidler, M. Seevers, and W. Zischank, "Magnetic fields and induced voltages in case of a direct strike-comparison of results obtained from measurements at a scaled building to those of IEC 62305-4," *J. Electrostatics*, vol. 65, pp. 379–385, 2007, (doi:10.1016/j.elstat.2006.09.004).

[14] W. Zischank, F. Heidler, J. Wiesinger, I. Metwally, A. Kern, and M. Seevers, "Laboratory simulation of direct lightning strokes to a modeled building: Measurement of magnetic fields and induced voltages," *J. Electrostatics*, vol. 60, pp. 223–232, 2004.

[15] Ph. Testé, T. Leblanc, F. Uhlig, and J. P. Chabrerie, "3D modeling of the heating of a metal sheet by a moving arc: Application to aircraft lightning protection," *Eur. Phys. J.*, vol. 11, pp. 197–204, 2000.

[16] C. A. Nucci, F. Rachidi, M. Ianoz, and C. Mazzetti, "Lightning-induced voltages on overhead lines," *IEEE Trans. Electromagn. Compat.*, vol. 35, no. 1, pp. 75–86, Feb. 1993.

[17] F. J. Heidler, M. Cvetic, and B.V. Stanic, "Calculation of lightning current parameters," *IEEE Trans. Power Del.*, vol. 4, no. 2, pp. 399–404, Apr. 1999.

[18] *Protection Against Lightning—Part 1: General Principles.* IEC 62305-1:2006, International Electrotechnical Commission, Geneva, Switzerland.

[19] *Protection Against Lightning. Part 3: Physical Damage to Structures and Life Hazard.* IEC 62305-3:2006, International Electrotechnical Commission, Geneva, Switzerland.

[20] Aircraft lightning environment and related test waveforms, SAE ARP 5412A, Feb. 21, 2005.

[21] KTA 2206:2000-06. "Auslegung von Kernkraftwerken gegen Blitzeinwirkugen," Sicherheitstechnische Regel des KTA.

[22] A. De Conti and S. Visacro, "Analytical representation of single-and double-peaked lightning current waveforms," *IEEE Trans. Electromagn. Compat.*, vol. 49, no. 2, pp. 448–451, May 2007.

[23] V. Javor and P. D. Rancic, "A channel-base current function for lightning return-stroke modeling," *IEEE Trans. Electromagn. Compat.*, vol. 53, no. 1, pp. 245–249, Feb. 2011 (doi: 10.1109/TEMC.2010.2066281).

[24] A. Andreotti, S. Falco, and L. Verolino, "Some integrals involving Heidler's lightning return stroke current expression," *Electr. Eng.*, vol. 87, no. 3, pp. 121–128 (doi: 10.1007/s00202-004-0240-8).

[25] F. Heidler and J. Cvetic, "A class of analytical functions to study the lightning effects associated with the current front," *Eur. Trans. Elect. Power*, vol. 12, no. 2, pp. 141–150, 2002.

[26] M. A. Uman, *The Lightning Discharge*. Orlando, FL: Academic Press, 1987, Appendix B3.

[27] K. Berger, R. B. Anderson, and H. Kroninger, "Parameters of lightning flashes," *Electra*, vol. 80, pp. 23–27, Jul. 1975.

[28] E. M. Thomson, M. A. Galib, M. A. Uman, W. H. Beasley, and M. J. Master, "Some features of stroke occurrence in Florida lightning flashes," *J. Geophys. Res.*, vol. 89, pp. 4910–4916, 1984.

[29] N. Cianos and E. T. Pierce, "A ground-lightning environment for engineering usage,", Stanford Research Institute Project 1834, Tech. Rep. 1, Stanford Research Institute, Menlo Park, CA, Aug. 1972.

[30] J. Schoene, M. A. Uman, V. A. Rakov, K. J. Rambo, J. Jerauld, C. T. Mata, A. G. Mata, D. M. Jordan, and G. H. Schnetzer, "Characterization of return stroke currents in rocket-triggered lightning," *J. Geophys. Res.*, vol. 14, pp. 1–9, 2009.

[31] P. Depasse, "Statistics on artificially triggered-lightning," *J. Geophys. Res.*, vol. 99, pp. 18515–18522, 1994.

[32] R. J. Fisher, G. H. Schnetzer, R. Thottappillil, V. A. Rakov, M. A. Uman, and J. D. Goldberg, "Parameters of triggered-lightning flashes in Florida and Alabama," *J. Geophys. Res.*, vol. 98, pp. 22887–22902, 1993.

[33] E. P. Krider, C. Leteinturier, and J. C. Willett, "Submicrosecond fields radiated during the onset of first return strokes in cloud-to-ground lightning," *J. Geophys. Res.*, vol. 101, pp. 1589–1597, 1996.

[34] J. Willett, E. Krider, and C. Leteinturier, "Submicrosecond field variations during the onset of first return strokes in cloud-to-ground lightning," *J. Geophys. Res.*, vol. 103, no. D8, pp. 9027–9034, 1998.

[35] V. Cooray, M. Fernando, C. Gomes, T. Sorensen, V. Scuka, and A. Pedersen, "The fine structure of positive return stroke radiation fields: A collaborative study between researchers from Sweden and Denmark," in *Proc. 24th Int. Conf. Lightning Protection*, Birmingham, U.K, 1998, pp. 78–82.

[36] C. Leteinturier, J. H. Hamelin, and A. Eybert-Berard, "Submicrosecond characteristics of lightning return-stroke currents," *IEEE Trans. Electromagn. Compat.*, vol. 33, no. 4, pp. 351–357, Nov. 1991.

[37] F. L. Pitts, R. A. Perala, T. H. Rudolph, and L. D. Lee, "New results for quantification of lightning/aircraft electrodynamics," *Electromagnetics*, vol. 7, pp. 451–485, 1987.

[38] J. C. Willett and E. P. Krider, "Rise times of impulsive high-current processes in cloud-to-ground lightning," *IEEE Trans. Antennas Propag.*, vol. 48, no. 9, pp. 1442–1451, Sep. 2000.

[39] J. Miyake, T. Suzuki, and K. Shinjou, "Characteristics of winter lightning current on Japan Sea Coast," *IEEE Trans. Power Del.*, vol. 1, no. 3, pp. 1450–1457, Jul. 1992.

[40] Y. Goto and K. Narita, “Electrical characteristics of winter lightning,” *J. Atmosph. Terr. Phys.*, vol. 12, pp. 57–64, 1995.

[41] J. Li, S. A. Cummer, W. A. Lyons, and T. E. Nelson, “Coordinated analysis of delayed sprites with high-speed images and remote electromagnetic fields,” *J. Geophys. Res.*, vol. 113, p. D20206 (doi:10.1029/2008JD010008, 2008).

[42] G. Lu, S. A. Cummer, J. Li, F. Han, R. J. Blakeslee, and H. J. Christian, “Charge transfer and in-cloud structure of large-charge-moment positive lightning strokes in a mesoscale convective system,” *Geophys. Res. Lett.*, vol. 36, p. L15805, 2009 (doi.10.1029/2009GL038880).

[43] L. Campos, M. Saba, O. Pinto, Jr., and M. Ballarotti, “Waveshapes of continuing currents for properties of M-components in natural negative cloud-to-ground lightning from high-speed video observations,” *Atmospheric Res.*, vol. 84, pp. 302–310, 2007.

[44] N. Kitagawa, M. Brook, and E. J. Workman, “Continuing currents in cloud-to-ground lightning discharges,” *J. Geophys. Res.*, vol. 67, pp. 637–647, 1962.

[45] M. M. F. Saba, O. Pinto, Jr., and M. G. Ballarotti, “Relation between lightning return stroke peak current and following continuing current,” *Geophys. Res. Lett.*, vol. 33, p. L23807, 2006 (doi:10.1029/2006GL027455).

[46] L. Campos, M. Saba, O. Pinto, Jr., and M. Ballarotti, “Waveshapes of continuing currents and properties of M-components in natural positive cloud-to-ground lightning,” *Atmospheric Res.*, vol. 91, pp. 416–424, 2009.

[47] V. A. Rakov and M. A. Uman, *Lightning: Physics and Effects*. New York: Cambridge Univ. Press, 2003, 687 p. See p. 222.

[48] J. Jerauld, M. A. Uman, V. A. Rakov, K. J. Rambo, and G. H. Schnetzer, “Insights into the ground attachment process of natural lightning gained from an unusual triggered-lightning stoke,” *J. Geophys. Res.*, vol. 112, p. D13113, 2007 (doi:10.1029/2006/JD007682).

[49] R. J. Fisher and M. A. Uman, “Recommended baseline direct-strike lightning environment for stockpile-to-target sequences,”, Tech. Rep. SAND-89-0192, Sandia National Laboratories, Albuquerque, NM, May 1989.

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